Analyses of the Performance of Liquid Dampers for Nutation in Spacecraft

K. AYACHE* AND R. LYNCH*
RCA Astro-Electronics Division, Princeton, N. J.

This paper deals with the performance of liquid-filled dampers for nutation damping of a spacecraft that uses a momentum wheel for attitude stabilization. The damper coupling factor is analyzed for toroidal and rectangular dampers completely filled with liquid, and for the same dampers containing a void, or bubble. Further analysis is made for a U-tube type of resonantly coupled liquid damper. In each case, the damper performance is evaluated relative to the completely filled toroidal liquid damper, illustrating the design regions wherein the damper effectiveness may be substantially improved by the presence of a void, or bubble, in the fluid path. Analysis of the performance of the U-tube resonant damper shows the potential for large gains in coupling factor, and increasingly better time-constant performance (the time constant being inversely proportional to the coupling factor).

Nomenclature

A = constant of integration
a,b = perpendicular distances from the center of the rectangle to the vertical and horizontal legs, respectively
A_t = area of the tube
D = sector length of the void
$F_{(r)}$ = a function of radius alone
$F_{(t)}$ = a function of time alone
f_D = viscous coupling factor of 2 dampers in quadrature
f_{D_s} = viscous coupling factor of a single damper
$\mathbf{H}$ = total spacecraft angular momentum vector
I = rotational inertia of the fluid
I_s = spin moment of inertia
I_T = spacecraft transverse moment of inertia
j = $(-1)^{1/2}$
J_0,J_1 = Bessel functions of order zero and one, respectively
K $\equiv$ $(w_0/w)^2\theta_0/\pi$
L = angular momentum of the fluid; L_1 and L_2 are the values in tubes of areas A_{t_1} and A_{t_2}, respectively
l = length of the U-shaped tube
l_s = distance from end of cap to the spin axis of U-shaped tube
P = pressure produced in the fluid by displacement of a void
R,r_0 = toroid and tube radii, respectively
V,V_A = fluid velocity, and velocity in tube of area A
V_{A_t} = fluid velocity in U-shaped tube
V_H,V_V = fluid velocities in horizontal and vertical legs; V_{H_0} and V_{V_0} are values at tube wall
V_0 = velocity of tube wall
V_H,V_V = volume flows of liquid in horizontal and vertical legs
∇P = pressure gradient in the fluid; ∇P_{H_0} and ∇P_{V_0} are values in horizontal and vertical legs, respectively
x = displacement of the viscous fluid in tube of area A_{t_1}
α $\equiv$ $2J_1(\beta_0)/\beta_0 J_0(\beta_0)$
β = complex argument of the Bessel function; β_0 = value at the tube wall [see Eq. (10)]
γ = half-cone angle of nutation
δ $\equiv$ $(a - b)/(a + b)$
θ_0 = half-angle intercepted by the void measured from the center of the toroid or rectangle
ρ,μ = fluid density and viscosity, respectively
ϕ = phase angle of the pressure reaction with respect to the exciting function
ω = exciting nutation frequency on fluid damper
ω_0 = damper spin rate along axis parallel to the spin axis
ω_s = spacecraft body spin rate
ω_{T_0} = amplitude of the transverse spin rate

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* Engineer, Engineering Department.

Introduction

THE gyroscopic effect of a spinning body is used to assist in establishing a stable attitude (or pointing direction) for a spacecraft. A momentum wheel—spinning about a central axis—provides equivalent stabilization. However, the free motion of the spacecraft also includes a wobble (nutation) of the spin axis, describing a cone around the total momentum vector. Nutation can be removed by the action of a properly installed energy-absorbing motion damper whose effect is a function of the amount of nutation still present. Recent models of one spacecraft utilize pairs of liquid-filled toroidal tubes, installed with the toroid axis at right angles to the spin axis (Fig. 1). The motion of nutation excites each toroid tangentially, causing the contained liquid, by virtue of the viscous effect (or drag) of the surrounding wall, to flow relative to the tube. The energy absorbed by this action is taken from the nutational forces, and some is transferred through the spacecraft to the spin axis.

A liquid-filled toroidal tube lends itself to reasonably straightforward analysis. However, since a (circular) toroid installation (Fig. 2a) may not always package efficiently with adjacent components, rectangular-shaped dampers (Fig. 2b) also are analyzed. The optimum points for comparative evaluation are established, and the performance characteristics are presented in terms useful to the design engineer for application to specific spacecraft. In considering practical devices, the effect of a void, or bubble, due to incomplete fill or thermal expansion also must be considered. A void can either improve or degrade damper performance. When a bubble damper is tuned to take advantage of liquid resonance effects, the performance can be improved by an order of magnitude over that of completely filled liquid dampers. The parametric relationships are developed in the analysis.

The five configurations analyzed, then, are: toroidal and rectangular, completely filled (Fig. 2); toroidal and rectangular with a void (Fig. 3); and tuned U-shape with a void (Fig. 4).

Toroidal Liquid Damper

The toroid (Fig. 2a), completely filled with a viscous liquid, is mounted on the spacecraft with the plane of the tube parallel to the spacecraft spin, or momentum, axis. (This may also be the flywheel axis.) When the vehicle is nutating or wobbling, the damper is subjected to various accelerations imposed by the motion of the spacecraft. Of all of the acceleration forces imposed on the completely contained liquid,

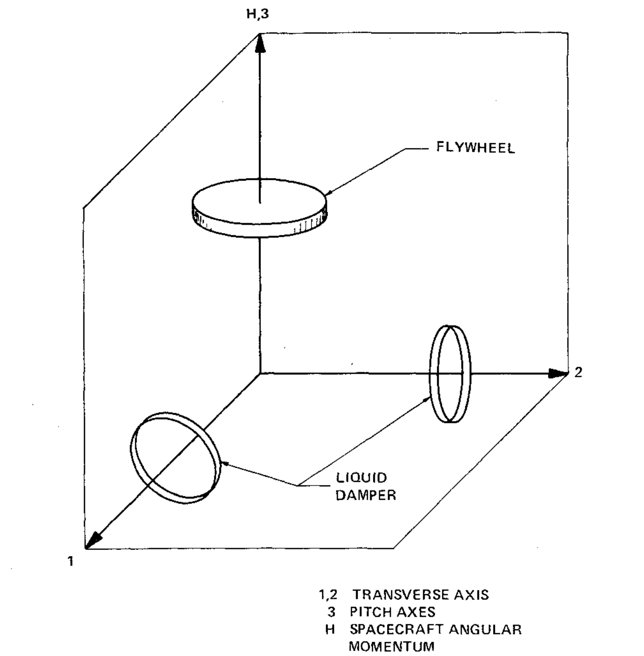

Fig. 1 Spacecraft axes, showing relation of angular momentum to dampers.

only those accelerations tangential to the long perimeter of the tube are effective in producing fluid motion. The non-tangential accelerations are balanced by pressure reactions within the fluid.

For purposes of this study, assume that the spacecraft possesses inertial symmetry about the spin, or flywheel, axis. The amplitude ω_{T_0} of the transverse spin under these conditions is a simple function of the half-cone angle of nutation γ and the total momentum vector **H**. Thus,

$$\omega_{T_0} = (|H| \sin\gamma)/I_T \tag{1}$$

where I_T is the transverse moment of inertia.

The exciting frequency applied to the damper, which is driven by the transverse motion of the vehicle, is a direct function of the difference between the spacecraft precessional spin and the body spin-rate parallel to the axis of symmetry. Thus, the radian frequency is

$$\omega = (|H| \cos\gamma)/I_T - \omega_s \tag{2}$$

where ω_s is the body spin rate.

Hence, for a simple spinning spacecraft,

$$\omega = (I_T^{-1} - I_s^{-1} \cos\gamma)|H| \tag{3}$$

where I_s is the spin inertia. On the other hand, for a flywheel-stabilized spacecraft in which the main body of the vehicle is despun to essentially zero speed,

$$\omega = (|H| \cos\gamma)/I_T \tag{4}$$

Therefore, for purposes of analysis, the exciting motion of the damper can be considered as a magnitude of ω_{T_0} at radian frequency ω.

The equation of fluid motion within the damper can be simply expressed in vector form,[1]

$$\rho d\mathbf{V}/dt = -\nabla\mathbf{P} + \mu\nabla^2\mathbf{V} \tag{5}$$

For a completely filled damper, it can be assumed that: 1) the velocity of the fluid is small, so that terms in V^2 can be neglected, 2) $r_0 \ll R$, so that the curvature R can be neglected, and 3) all body forces are cancelled by internal fluid pressure reactions except for those producing longitudinal fluid shear. The equation of motion therefore becomes

$$\rho d\mathbf{V}/dt = \mu\nabla^2\mathbf{V} \tag{6}$$

Taking the solution as a product of a function of time alone and a function of radius alone, the motion equation becomes as follows:

$$\rho F'(t)F(r) = \mu[F''(r) + (1/r)F'(r)]F(t) \tag{7}$$

This simplifies to the relationship

$$F''(r) + (1/r)F'(r) - (\rho/\mu)[F'(t)/F(t)]F(r) = 0 \tag{8}$$

Taking now the excitation as a function of time in complex form, $F(t) = e^{-j\omega t}$, we obtain

$$F''(r) + (1/r)F'(r) + j\omega(\rho/\mu)F(r) = 0 \tag{9}$$

This equation has for its solution a Bessel function of complex argument

$$F(r) = AJ_0(\beta) \tag{10}$$

where $\beta \equiv r(j\rho\omega/\mu)^{1/2}$; we also define $\beta_0 \equiv r_0(j\rho\omega/\mu)^{1/2}$.

Since, all the wall of the tube, the fluid has the same velocity as the tube wall, the constant A may be calculated. The velocity of the tube wall is $R\omega_{T_0} = V_0$. Hence, $F(r) = V_0 J_0(\beta)/J_0(\beta_0)$.

The fluid velocity therefore may be written as follows:

$$V = F(r)F(t) = V_0[J_0(\beta)/J_0(\beta_0)]e^{-j\omega t} \tag{11}$$

From this equation, the angular momentum may be calculated by integration over the tube volume

$$L = \int_0^{r_0} (2\pi R)\rho 2\pi r dr V R$$

$$L = \rho 2\pi R^2 \pi r_0^2 V_0 \alpha e^{-j\omega t} \tag{13}$$

where

$$\alpha \equiv 2J_1(\beta_0)/\beta_0 J_0(\beta_0) \tag{14}$$

If the rotational inertia of the fluid is expressed as if it were solid mass affixed to the tube wall, the inertia can be written a

$$I = \rho 2\pi R \pi r_0^2 R^2 \tag{15}$$

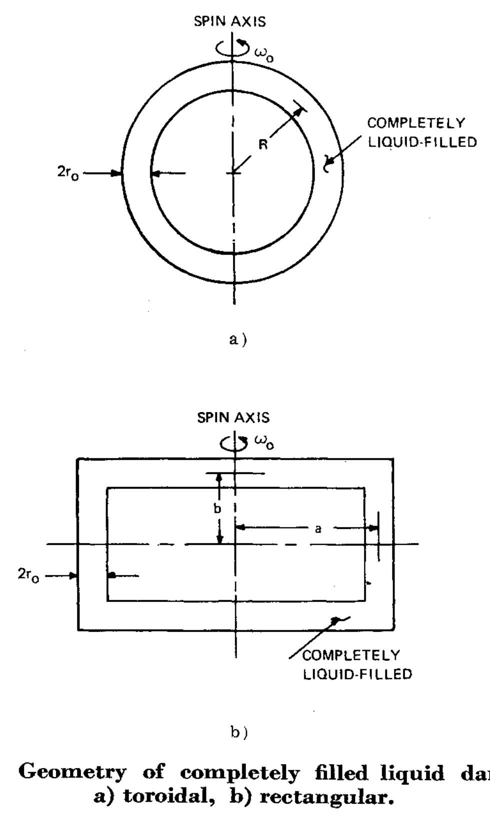

Fig. 2 Geometry of completely filled liquid dampers: a) toroidal, b) rectangular.

Substituting this I and $V_0 = \omega_{T_0}R$ into (13),

$$L = I\omega_{T_0}\alpha e^{-j\omega t} \tag{16}$$

A tabulation of α [Eq. (14)] may be found in Jahke and Emide.[2] It is a dimensionless, complex number and a function of the argument β_0. The real part represents that fraction of the angular momentum in phase with the tube wall and is therefore ineffective in damping out the precessional motion. The imaginary part, however, is that portion of the fluid momentum which is in a 90° phase relation with the motion of the tube wall. The latter represents the effective component of the fluid momentum for spacecraft damping. Therefore, the magnitude of the imaginary part of α is the effectiveness, or viscous coupling, factor for the damper pair

$$f_D = \text{imaginary } [\alpha] \tag{17}$$

$$f_{D_s} = f_D/2 \tag{18}$$

This f_{D_s} is plotted vs $r_0\ (\omega\rho/\mu)^{1/2}$ for $K = 0$ in Fig. 5.

Rectangular Liquid Damper

For the rectangular-tube damper (Fig. 2b), the basic assumptions taken in the analysis of the toroidal damper apply, with the following additions and restrictions. The internal pressure balances the acceleration forces as before, except in the longitudinal directions. Since the restriction of fluid flow differs in the vertical and horizontal legs, longitudinal pressure will also develop. Furthermore, fluid flow around the corners (provided with generous bends) normally develops vortex type velocities transverse to the tube. However, these velocities are considered sufficiently small that their effects are neglected. Therefore, the equation of motion of the fluid in the tube includes a pressure term and will conform to Eq. (5). The flow in each leg (horizontal and vertical) is treated as dynamically induced and further modified by the pressure effects arising from the differing flow restrictions in the legs of the damper. The phase and magnitude of the pressure reaction are computed from conditions of continuity of flow and from the requirement that the total pressure drop around the damper must be zero. In the horizontal leg, following the example of the toroidal damper analysis,

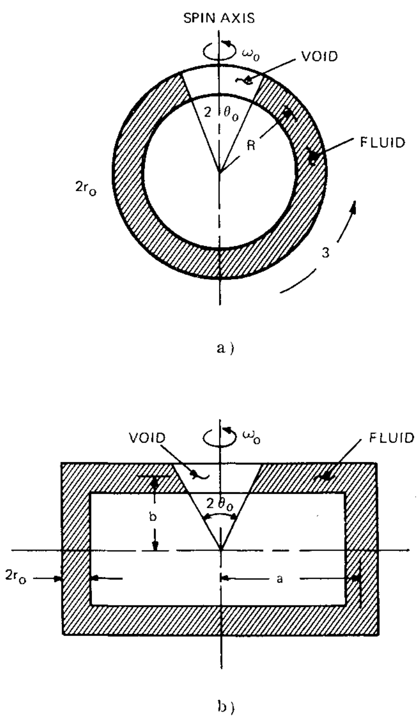

Fig. 3 Geometry of liquid dampers with void: a) toroidal, b) rectangular.

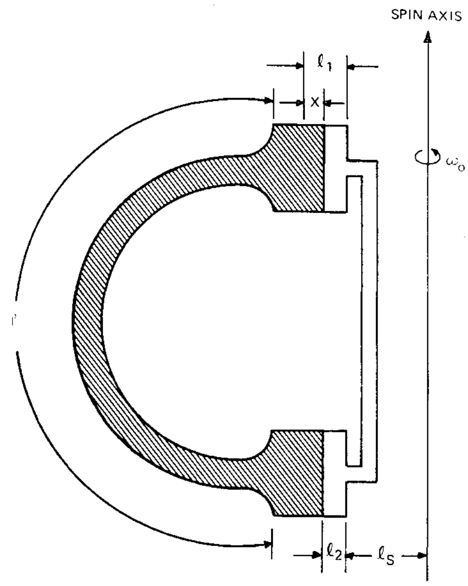

Fig. 4 Geometry of a tuned, U-shaped liquid damper with a void.

$$V_{H_0} = \omega_{T_0}b$$

Let $V = F(t)F(r)$ be a solution. Therefore,

$$V = [AJ_0(\beta) + \xi]e^{-j\omega t} \tag{19}$$

where $\xi \equiv \nabla P_0|H|e^{j\phi}/j\omega\rho$. The constant A can be evaluated from the condition that

$$V_H = V_0 \quad \text{at} \quad r = r_0 \quad \therefore A = V_0 - \xi/J_0(\beta_0) \tag{20}$$

Thus, the fluid velocity in the horizontal leg may be expressed in full as follows:

$$V_H = [V_{H_0} - \xi_0 J_0(\beta)/J_0(\beta_0) + \xi_0]e^{-j\omega t} \tag{21}$$

where $\xi_0 \equiv \nabla P_{H_0}e^{j\phi}/j\omega\rho$. A similar equation may be developed for the vertical leg

$$V_V = [V_{V_0} - \zeta_0 J_0(\beta)/J_0(\beta_0) + \zeta_0]e^{-j\omega t} \tag{22}$$

where $\zeta_0 \equiv \nabla P_{V_0}e^{j\phi}/j\omega\rho$.

To preserve the condition that the pressure drop around the damper shall sum to zero, the pressure rise in the vertical leg must equal the pressure fall in the horizontal leg. Since ∇P_{H_0} represents the longitudinal pressure gradient, this condition may be expressed algebraically as follows:

$$2a\nabla P_{H_0} = -2b\nabla P_{V_0} \tag{23}$$

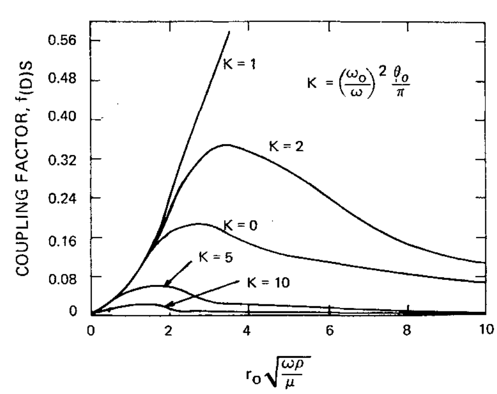

Fig. 5 Viscous coupling factors of a toroidal damper for different size bubbles.

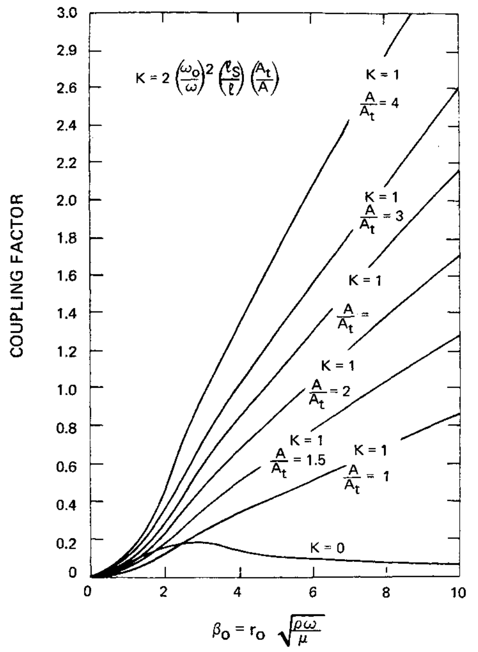

Fig. 6 Viscous coupling factors of U-shaped damper for different size bubbles.

It is further required, because of continuity of flow from one leg to the other, that the volume flow be identical in each leg; i.e., $V_H = V_V$. Thus, noting that the volume flow relative to the tube is required, the equating of these two volume flows yields

$$\int_0^{r_0} V_H 2\pi r dr - V_{H_0}\pi r_0^2 = \int_0^{r_0} V_V 2\pi r dr - V_{V_0}\pi r_0^2 \quad (24)$$

$$(\nabla P_{H_0} - \Delta P_{V_0})e^{j\phi} = (V_{H_0} - V_{V_0})j\omega\rho \quad (25)$$

Since $V_{H_0} = \omega_{T_0} b$, $V_{V_0} = \omega T_0 a$, and $a\nabla P_{H_0} = -b\nabla P_{V_0}$, Eq. (25) can be modified to

$$\nabla P_{H_0} e^{j\phi} = -\omega_{T_0} b\delta j\omega t \quad (26)$$

where

$$\delta \equiv (a - b)/(a + b) \quad (27)$$

We may now collect the pieces of analysis in the full equation for fluid velocity in the horizontal legs and in the vertical legs. Thus,

$$V_H = \omega_{T_0} b[(1 + \delta)J_0(\beta)/J_0(\beta_0) - \delta]e^{-j\omega t} \quad (28a)$$

and

$$V_V = \omega_{T_0} a[(1 - \delta)J_0(\beta)/J_0(\beta_0) + \delta]e^{-j\omega t} \quad (28b)$$

Now, to compare the angular momentum of the total fluid, the angular momentum due to the two horizontal legs is added to that of the vertical legs. The result is

$$L = I_R\omega_{T_0}[(1 - \delta^2)\alpha + \delta^2]e^{-j\omega t} \quad (29)$$

where

$$I_R \equiv 4ab(a + b)\pi r_0^2\rho \quad (30)$$

can be thought of as an inertia term formed by multiplying the mass of the fluid within the damper by the product of rectilinear dimension. Hence, as with the toroidal damper analysis, the effectiveness factor is the imaginary part of the bracketed term in Eq. (29). Inspection shows that the rectangular damper effectiveness factor is reduced from that of the toroidal type

$$f_{D_{s(\text{rect})}} = (1 - \delta^2)f_{D_{s(\text{toroid})}} \quad (31)$$

Note that the rectangular damper performance uses the rectilinear inertia I_R as defined earlier. Table 1 compares relative performances of toroidal and rectangular dampers on the basis of the same mass of fluid in each damper type.

The centrifugal effects of a bubble (Fig. 3a) impose pressure forces on the fluid motion, thereby changing the fundamental characteristics of the damper assembly. The dynamic effects arise from the sharp density change at the liquid surface. The forces can attenuate or enhance the damping action of the assembly. The bubble is taken always as a void across the complete tube area. Surface tension effects are not considered. [These tensions can be significant in affecting damper performance at low spacecraft spin rates, as can temperature (hence density) gradients along the fluid path; here we consider only a void, i.e., a discontinuous change in fluid density.]

The pressure produced in the fluid by a displacement of a void (of sector length D) in the tube from its position of equilibrium when the damper is spinning is expressed as

$$P = \int_{R(\theta_0+D\theta)}^{R\theta_0} D\omega_0^2\rho dD + \int_{R\theta_0}^{R(\theta_0+D\theta)} D\omega_0^2\rho dD \quad (32)$$

$$P = 2\omega_0^2R^2\rho\theta_0\nabla\theta \quad (33)$$

The pressure gradient is $\nabla P = P/2\pi R$. For a small $\Delta\theta$, $D = R\Delta\theta$, and $\Delta P = \omega_0^2\rho\theta_0 D/\pi$. Again solving Eq. (5), let $\nabla P = F(t)$ and $V = f(r)\cdot f(t)$ be a product solution. Substituting into the equation of motion

$$\rho f(r)f'(t) = -F(t) + \mu[f''(r) + (1/r)f'(r)]f(t) \quad (34)$$

Let $f(t) = e^{-j\omega t}$. The differential equation becomes

$$\frac{1}{\mu}\frac{F(t)}{e^{-j\omega t}} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\rho}{\mu}Vj\omega \quad (35)$$

Note that $V = f(t)\cdot f(r)$ is a solution, and

$$\int_0^{r_0} rVdr$$

is stationary and is multiplied by a time function. Hence,

$$\int_0^{r} rVdr$$

has a particular stationary value. From consideration of the ∇P and the displacement D, one may relate the average fluid velocity and the rate of change of ∇P

$$d(\nabla P/dt) = d(cD/dt) \quad (36)$$

where

$$c \equiv \rho\omega_0^2\theta_0/\pi \quad (37)$$

Then

$$\nabla P = \left(\frac{2}{r_0^2}\int_0^{r_0} rVdr - V_0\right)\frac{ce^{-j\omega t}}{(-j\omega)} \quad (38)$$

Equation (5) becomes

$$\nabla P = (\lambda - V_0c)e^{-j\omega t}/(-j\omega) \quad (39)$$

Table 1 Relative performances of dampers

Configuration	$I_{(D)}$	f_s
Circular	1	1
Rectangular, $a/b = 1$	0.616	1
$a/b = 2$	0.547	$\frac{8}{9}$
$a/b = 3$	0.461	$\frac{3}{4}$
$a/b = 4$	0.394	$\frac{16}{25}$

where

$$\lambda \equiv \frac{2c}{r_0^2} \int_0^{r_0} rVdr \tag{40}$$

$$\left(\frac{\lambda - V_0 c}{\mu}\right) \frac{1}{-j\omega} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} - \frac{\rho}{\mu}(-j\omega)V \tag{41}$$

which has a standard form, and its solution is

$$V = AJ_0(\beta) - \psi \tag{42}$$

where

$$\psi \equiv (\lambda - V_0 c)/(-j\omega^2)\rho \tag{43}$$

The initial conditions are

$$V = V_0 \quad \text{at} \quad r = r_0$$

or

$$A = (V_0 + \psi)/J_0(\beta_0) \tag{44}$$

The general solution of Eq. (41) which satisfies the boundary conditions is

$$V = [(V_0 + \psi)J_0(\beta)/J_0(\beta_0) - \psi]e^{-j\omega t} \tag{45}$$

Let us evaluate λ as follows. Substituting the velocity given in Eq. (45) into Eq. (40) and integrating, we obtain

$$\lambda = cV_0 \frac{\alpha + (1-\alpha)c/(-j\omega)^2\rho}{1 + (1-\alpha)c/(-j\omega)^2\rho} \tag{46}$$

where α is given by (14).

The angular momentum is

$$L = \int_0^{r_0} \mathbf{R} \times m\mathbf{V} = R^2(2\pi)^2\rho \int_0^{r_0} rVdr \tag{47}$$

Substituting Eq. (45) and integrating,

$$L = 2(\pi r_0 R)^2\rho[(V_0 + \psi)\alpha - \psi]e^{-j\omega t} \tag{48}$$

Now, by substituting the original expressions for ψ [Eq. (43)] and then c [Eq. (37)] into (48), and by assuming a solid moment of inertia I,

$$I = (2\pi r_0^2)(2\pi R_\rho)R^2/2 \tag{49}$$

and letting

$$K \equiv (\omega_0/\omega)^2\theta_0/\pi \tag{50}$$

we simplify Eq. (48) to

$$L = \frac{IV_0}{R}\frac{\alpha + K(\alpha - 1)}{1 + K(\alpha - 1)} e^{-j\omega t} \tag{51}$$

The effectiveness factor is

$$f_{D_s} = \tfrac{1}{2}\,\text{imaginary}\,\{[\alpha + K(\alpha-1)]/[1 + K(\alpha - 1)]\} \tag{52}$$

Since α is a complex, let

$$\alpha = \alpha_{\text{real}} + j\alpha_{\text{imag}} \equiv 2J_1(\beta_0)/\beta_0 J_0(\beta_0) \tag{53}$$

$$|\alpha| = [\alpha^2_{\text{real}} + \alpha^2_{\text{imag}}]^{1/2} \tag{54}$$

Then, for a single toroidal damper with a bubble

$$f_{D_s} = \tfrac{1}{2}\, i\alpha_{\text{imag}}/[(1-K)^2 + K^2|\alpha|^2 + 2K(1-K)\alpha_{\text{real}}] \tag{55}$$

Tuned, U-Shaped Liquid Damper with a Bubble

Figure 4 illustrates schematically a possible tuned damper for a rotating vehicle. Tuning is accomplished by selecting the ratio of the cross-sectional areas of the curved main damper tube and its expanded end caps. The end caps are connected by a small vapor line to prevent vapor lock. (This line may not be theoretically necessary if only liquid vapor is present.) Let A_t and l be the area and length of the tube; l_s distance from an end cap to the spin axis; and x, the displacement of the viscous fluid in the tube. The pressure gradient produced in the tube by a displacement x of the viscous fluid in the tube when the damper is spinning is

$$\nabla P = P/l = 2\rho(l_s/l)\omega_0^2 x \tag{56}$$

$$\frac{d\nabla P}{dt} = 2\rho\omega_0^2 \frac{l_s}{l}\frac{1}{A_t}\int_0^{r_0} 2\pi rVdr \tag{57}$$

Since the tube is a toroid, the basic assumptions of the analysis of a toroidal liquid damper apply here also. If we compare Eq. (58) with Eq. (38), we find

$$2/r_0^2 = 4\pi/A_t \quad \text{and} \quad c = \rho\omega_0^2\, l_s/l \tag{58}$$

The equation analogous to (46) becomes

$$\lambda = cV_0 \frac{\alpha - (1-\alpha)c/\rho\omega^2}{1 - (1-\alpha)2A_t c/\rho\omega^2 A}\,\frac{2A_t}{A} \tag{59}$$

Let L_1 and L_2 be the angular momenta in tubes of areas A_{t_1} and A_{t_2}, respectively. The total angular momentum is $L_T = L_1 + L_2$. Then, since

$$dL_1 = \rho 2\pi r dr l R V_{A_t} \tag{60}$$

and

$$L_2 = R\rho A_t(l_1 + l_2)V_A \tag{61}$$

where

$$V_A = \frac{1}{A}\int_0^{r_0} 2\pi r V_{A_t} dr \tag{62}$$

Thus,

$$L_T = 2\pi R\rho(l + l_1 + l_2)\int_0^{r_0} rV_{A_t}dr \tag{63}$$

where V_{A_t} is given by Eq. (45). By steps similar to those in the previous sections, we find

$$L_T = \frac{IV_0}{R}\left[\frac{2\alpha - (KA/A_t)(1-\alpha)}{2 - 2K(1-\alpha)}\right]e^{-j\omega t} \tag{64}$$

where

$$I \equiv \rho R^2 A_t(l + l_1 + l_2) \tag{65}$$

and

$$K \equiv 2(A_t/A)(\omega_0/\omega)^2 l_s/l \tag{66}$$

and the effectiveness, or viscosity coupling, factor is equal to

$$f_{D_s} = \frac{1}{2}\,\text{imaginary}\left[\frac{2\alpha - (KA/A_t)(1-\alpha)}{2 - 2K(1-\alpha)}\right] \tag{67}$$

A plot for f_{D_s} is shown in Fig. 6.

References

[1] Shamer, I. H., *Mechanics of Fluids*, McGraw-Hill, New York, 1962, p. 279.

[2] Jahke, E. and Emide, F., *Tables of Functions*, 4th ed., Dover, New York, 1945, p. 266.